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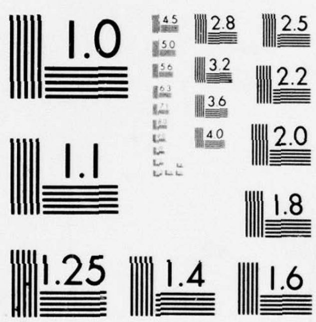

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THE ACCURACY OF GRAVIMETRIC DEFLECTIONS OF THE VERTICAL AS DERIVED FROM THE GEM 7 POTENTIAL COEFFICIENTS AND TERRESTRIAL GRAVITY DATA

Lars Sjöberg

The Ohio State University
Research Foundation
Columbus, Ohio 43212

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The deflections of the vertical can be determined with a combined method using point gravity anomalies, mean anomalies and satellite spherical harmonics in the inner-, intermediate- and remote zones, respectively. In essential, the error sources of the intermediate and distant zone are studied.

The RMS error contribution of the remote zone was found considerably → next page

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dependent on the distance of truncation. Another dominating error source is the error due to lack of more detailed gravity data than $1^\circ \times 1^\circ$ mean anomalies in the intermediate zone. Smaller block sizes are recommended between the distances 1° and 10° from the point of computation.

deg deg deg

A significant gain of accuracy will be achieved in the total error by extending the radius of truncation to at least 30°. Anticipating an inner zone error of $1''.8$ (a solution by collocation) the total RMS error will hardly be less than $2''.2$, while for an inner zone error of $0''.4$, the total error might decrease to $1''.3$.

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Foreword

This report was prepared by Lars Sjöberg, Research Associate, Department of Geodetic Science, The Ohio State University, under Air Force Contract No. F19628-76-C-0010, The Ohio State University Research Foundation Project No. 710335, Project Supervisor, Richard H. Rapp. The contract covering this research is administered by the Air Force Geophysics Laboratory, L. G. Hanscom Air Force Base, Massachusetts, with Mr. Bela Szabo, Contract Monitor.

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Table of Contents

1. Introduction

Recently, combined methods have been introduced for the determination of the deflections of the vertical. Those methods may, first of all, incorporate the following data: satellite derived potential coefficients, mean gravity anomalies and point gravity observations. The idea is that the inner zone is determined in detail from the point anomalies, for instance, by using the method of least-squares collocation, while the effect of an intermediate zone is calculated with the Vening Meinesz' formula using mean gravity anomalies, and finally, the remote zone contribution is represented by a spherical harmonic expansion.

In this paper we are going to study the errors of the combined method. The investigation is mainly restricted to the errors of the intermediate and remote zones. The error contribution of the inner zone differs being much dependent on the method of computation and density and quality of the data. For this zone, we will adopt some representative values found in the literature.

2. Computational Method

We briefly summarize the combined method presented in Lachapelle (1977). The components ξ and η are considered to consist of three subcomponents (ξ_0, ξ_1, ξ_2; η_0, η_1, η_2) such that:

(1)
$$\xi = \xi_0 + \xi_1 + \xi_2$$
and
$$\eta = \eta_0 + \eta_1 + \eta_2$$

Each of the subcomponents is the contribution of the deflections from a specific zone around the point of computation (see Figure 1). Exceptions are ξ_0 and η_0, which also serve as "reference field" in σ_1 and σ_2.

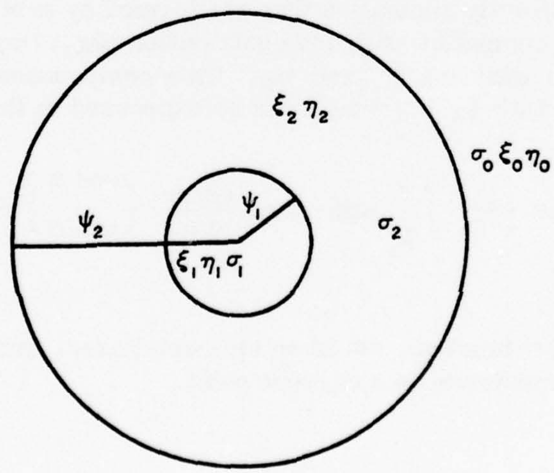

Figure 1. Subdivision of the Surface Around the Point of Computation into Inner Zone (σ_1), Intermediate Zone (σ_2) and Remote Zone (σ_0).

The values of ξ_0 and η_0 are computed from the fully normalized potential coefficients $\overline{J}^*_{nm}$ and $\overline{K}_{nm}$ in the following way (Lachapelle, ibid., p. 3):

$$(2) \qquad \begin{Bmatrix} \xi_0 \\ \eta_0 \end{Bmatrix} = - \sum_{n=2}^{n_{max}} \left(\frac{r_B}{r}\right)^{n+1} \sum_{m=0}^{n} \left[\overline{J}^*_{nm} \begin{Bmatrix} D\varphi \\ D\lambda \end{Bmatrix} \overline{R}_{nm}(\varphi,\lambda) + \overline{K}_{nm} \begin{Bmatrix} D\varphi \\ D\lambda \end{Bmatrix} \overline{S}_{nm}(\varphi,\lambda) \right]$$

where n_{max} is the maximum degree of expansion, and:

$$D\varphi \begin{Bmatrix} \overline{R}_{nm}(\varphi,\lambda) \\ \overline{S}_{nm}(\varphi,\lambda) \end{Bmatrix} = \begin{Bmatrix} \cos m\lambda \\ \sin m\lambda \end{Bmatrix} \frac{d}{d\varphi} \overline{P}_{nm}(\sin\varphi)$$

$$D\lambda \begin{Bmatrix} \overline{R}_{nm}(\varphi,\lambda) \\ \overline{S}_{nm}(\varphi,\lambda) \end{Bmatrix} = \begin{Bmatrix} -\sin m\lambda \\ \cos m\lambda \end{Bmatrix} \overline{P}_{nm}(\sin\varphi) \frac{m}{\cos\varphi}$$

r_B/r = ratio between the radius (r_B) of the internal sphere (to which $\overline{J}^*_{nm}$ and $\overline{K}_{nm}$ are referred) and the radius (r) of the point of computation.

Remark 1. Formula (2) is a slight generalization of Lachapelle's formula for points at an arbitrary height above the sphere to which the coefficients $\overline{J}^*_{nm}$ and $\overline{K}_{nm}$ refer. For low degree expansions at the surface of the earth $r_B/r = 1$ is a good approximation.

The values of ξ_2 and η_2 are obtained by applying Vening Meinesz' integral formula to gravity anomalies that are formed by subtracting from mean terrestrial free-air anomalies ($\Delta\overline{g}$) the contribution (Δg_s) implied by the potential coefficients used in computing ξ_0 and η_0. This computation, which is carried out for the zone σ_2 with $\psi_1 \le \psi \le \psi_2$, can be expressed in the following way:

$$(3) \qquad \begin{Bmatrix} \xi_2 \\ \eta_2 \end{Bmatrix} = \frac{1}{4\pi G} \iint_{\sigma_2} (\Delta\overline{g} - \Delta g_s) \frac{dS(\psi)}{d\psi} \begin{Bmatrix} \cos\alpha \\ \sin\alpha \end{Bmatrix} d\sigma$$

where $S(\psi)$ is Stokes' function, $d\sigma$ is an elemental area, and α is the azimuth from the point of computation to a current point.

The classical Vening Meinesz' formula is valid strictly only for a sphere. However, terrestrial data $\Delta\bar{g}$ may be used provided that either they are continued analytically to the internal sphere or that a terrain correction (Molodenskii term) is applied to $\Delta\bar{g}$ (Heiskanen and Moritz, 1967, pp. 315 and 313, respectively).

Lachapelle (ibid.) used $1^\circ \times 1^\circ$ mean gravity anomalies for these computations, which were extended to $\psi_2 = 8^\circ$. ψ_1 varied between $0\overset{\circ}{.}7$ and $1\overset{\circ}{.}5$, depending on the density of the point gravity data in the inner zone.

The values of ξ_1 and η_1 may be obtained in different ways; Lachapelle (ibid.) used least squares collocation. Such an approach has the advantage that there is no difficulty to compute the effect of the innermost zone and we can easily incorporate heterogeneous data in the computations. As pointed out by Tscherning (1974) a considerable gain in accuracy will be achieved if we include astrogeodetic deflection of the vertical in the set of observations. On the other hand, we have to limit the number of observations to a few hundred in order to keep the computer time at a reasonable level. Another difficulty is the instability that occurs if two observations are located close to each other. The original Vening Meinesz' formula does not suffer from these limitations.

3. Error Analysis

We now study the error propagation due to the data and other sources. It is assumed that the numerical integration of Vening Meinesz' formula is performed with such an accuracy that the integration errors can be neglected. Furthermore, the model errors caused by disregarding the flattening of the earth (in the order of 3×10^{-3}) are not considered.

The gravity data are assumed to be corrected for the terrain effect in Vening Meinesz' formula. If this correction is omitted, an additional error in the order of $0\overset{''}{.}2$, is introduced (Moritz, 1966; Dimitrijevich, 1972).

The effect of the earth's atmosphere can be estimated in the following way (Moritz, 1974). A constant distribution of air is assumed above the reference ellipsoid. This homogeneous mass has no effect on the deflection. Next, the fictitious density of the atmosphere inside the topography (above the reference ellipsoid) is subtracted from the density of the topography. (This modified density of the topography can be used for a simultaneous correction for terrain and atmosphere.) We obtain the following contribution from each compartment to the correction for the atmosphere:

$$\text{atm. corr.} = - \text{terrain corr.} \times \frac{\text{density of atmosphere}}{\text{density of topography}}$$

As the terrain correction is in the order 0".2 it is obvious that the atmosphere has no practical effect on the deflections of the vertical.

Finally, we regard the earth constants as known (without errors).

3.1 Errors from the Remote Zone

The error contribution of the remote zone is dependent on the spherical distance ψ_2 (see Figure 1). A formula for the RMS influence of his zone on the total deflection of the vertical:

$$\theta = \sqrt{\xi^2 + \eta^2} \tag{4}$$

is given in Heiskanen and Moritz (1967, p. 262). However, as pointed out by deWitte(1966) and Hagiwara (1972), Molodenskii's truncation coefficients, Q_n, in this formula need to be modified. A generalization is obtained by substituting c_n by $(r_B/r)^{2(n+2)}\, c_n$ (cf. Remark 1). With these modifications, the mean square contribution of the remote zone becomes:

$$\delta\theta^2 = \frac{1}{4G^2} \sum_{n=2}^{\infty} n\,(n+1)\, \bar{Q}_n^2\,(\psi_2) \left(\frac{r_B}{r}\right)^{2(n+1)} c_n \tag{5}$$

where

c_n	=	anomaly degree variances at the internal (Bjerhammar) sphere
$\bar{Q}_n(\psi_2)$	=	$Q_n\,(\psi_2) + S\,(\psi_2)\, P_n\,(\cos\psi_2)\, \sin\psi_2 \,/\, n(n+1)$
$S\,(\psi_2)$	=	Stokes' function
ψ_2	=	geocentric distance of truncation

As the spherical harmonic expansion for a given set of coefficients can be regarded as a reference field in the inner and intermediate zones, errors in the coefficients will influence the remote zone contributions only. Thus, the errors of the remote zones are of two kinds: the potential coefficient errors and the truncation errors. From (2) and (5) we obtain the following mean square propagation of <u>the potential coefficient errors:</u>

$$(6) \qquad \delta\theta^2_{o,1} = \frac{1}{4G^2} \sum_{n=2}^{n_{max}} n\,(n+1)\,\bar{Q}^2_n(\psi_2) \left(\frac{r_B}{r}\right)^{2(n+2)} dc_n$$

where dc_n is the mean square error of c_n. This error can be determined in the following way.

Let us expand the gravity anomaly (Δg^*) at the internal sphere ($r_B = r$) into a series of spherical harmonics to degree n_{max} [cf. (2)]:

$$(7) \qquad \Delta g^* = \sum_{n=2}^{n_{max}} \Delta g^*_n$$

where

$$\Delta g^*_n = G\,(n-1) \sum_{m=0}^{n} [\bar{J}^*_{nm}\,\bar{R}_{nm}(\varphi,\lambda) + \bar{K}_{nm}\,\bar{S}_{nm}(\varphi,\lambda)]$$

Thus, the coefficient errors $\delta\bar{J}_{nm}$ and $\delta\bar{K}_{nm}$ are propagated according to:

$$\delta\Delta g^*_n = G\,(n-1) \sum_{m=0}^{n} [\delta\bar{J}_{nm}\,\bar{R}_{nm}(\varphi,\lambda) + \delta\bar{K}_{nm}\,\bar{S}_{nm}(\varphi,\lambda)]$$

and the global mean square error of c_n is finally given by:

$$(8) \qquad dc_n = M\{\delta\Delta g^{*2}_n\} = G^2(n-1)^2 \sum_{m=0}^{n} (\delta\bar{J}^2_{nm} + \delta\bar{K}^2_{nm})$$

where $M\{x\}$ is the global average of x:

$$M\{x\} = \frac{1}{4\pi} \iint_\sigma x \, d\sigma$$

In (8) we have used the orthogonality property of the spherical harmonics over a sphere. For a different estimate of δc_n, see Rapp (1973). In Table 1 we give the mean square errors δc_n computed from the errors of the GEM 7 coefficients (Wagner, 1976).

Table 1. Mean Square Errors of the Degree Variances for GEM 7 from Wagner (1976, Table 27). Fully Normalized Harmonics. G = 980 gal.

n	$\delta\sigma_n^2\ 10^{18}$ *	δc_n [mgal^2]†
2	35	0.0000
3	357	0.0014
4	219	0.0019
5	938	0.0144
6	715	0.0172
7	2103	0.0727
8	1710	0.0805
9	3368	0.2070
10	3191	0.2503
11	5990	0.5753
12	4924	0.5722
13	7865	1.0877
14	7247	1.1762
15	11271	2.1216
16	11357	2.4541

$$* \quad \delta\sigma_n^2 = \sum_{m=0}^{n} (\delta\bar{J}_{nm}^2 + \delta\bar{K}_{nm}^2)$$

$$\dagger \quad \delta c_n = G^2 (n-1)^2\ \delta\sigma^2$$

Finally, by inserting these values for δc_n into formula (6), the potential coefficient error for $n_{max} = 16$ was determined. The ratio r_B/r was set equal to 1 in this low degree expansion. The result is depicted in Figure 2.

The other error source of the remote zone, the error due to the truncation of the spherical harmonic expansion, is also given by formula (5). The mean square value of the truncation error is:

$$\delta\theta_{0,2}^2 = \frac{1}{4G^2} \sum_{n_{max}+1}^{\infty} n(n+1)\ \bar{Q}_n^2(\psi_2) \left(\frac{r_B}{r}\right)^{2(n+2)} c_n \qquad (9)$$

For the numerical computation of this error, we use the degree variances c_n of Tscherning and Rapp (1974):

$$c_n = A\,(n-1)/(n-2)\,(n+24)\,, \quad n \geq 3 \tag{10}$$

$$s = (r_B/r)^2 = 0.999617$$

$$A = 425.28 \text{ mgal}^2$$

The convergence of (9) is very slow for ψ_2 close to 0 while for larger angles, say $\psi_2 \geq 5°$, it converges well. Formula (9) is illustrated in Figure 2. The excellent subroutine of Paul (1973) was used for a rapid determination of Q_n, and the series was truncated at $n = 2000$. A remarkable minimum (0."06) of the RMS error is obtained for $\psi_2 = 40°$.

3.2 Errors Due to Lack of More Detailed Data in σ_2

The application of Vening Meinesz' formula (3) for the determination of ξ_2 and η_2 requires, theoretically, that Δg is known at each point in σ_2. In practice we limit ourselves to using mean gravity anomalies in this area. We shall now estimate the error due to the lack of more detailed gravity material in σ_2. The data is assumed to be located on a mean earth sphere of radius r.

Let us expand Δg into Laplace harmonics:

$$\Delta g = \sum_{n=0}^{\infty} \Delta g_n$$

The mean gravity field $(\overline{\Delta g})$ is related to Δg according to:

$$\overline{\Delta g}\,(y) = \iint_{\sigma} B\,(y \cdot x)\,\Delta g\, d\sigma = \sum_{n=2}^{\infty} \beta_n\, \Delta g_n$$

where $B\,(y \cdot x)$ is the averaging operator and β_n are its eigen values. If we approximate each block of mean anomalies with a circular cap with equal area we obtain (see Meissl, 1971, p. 24):

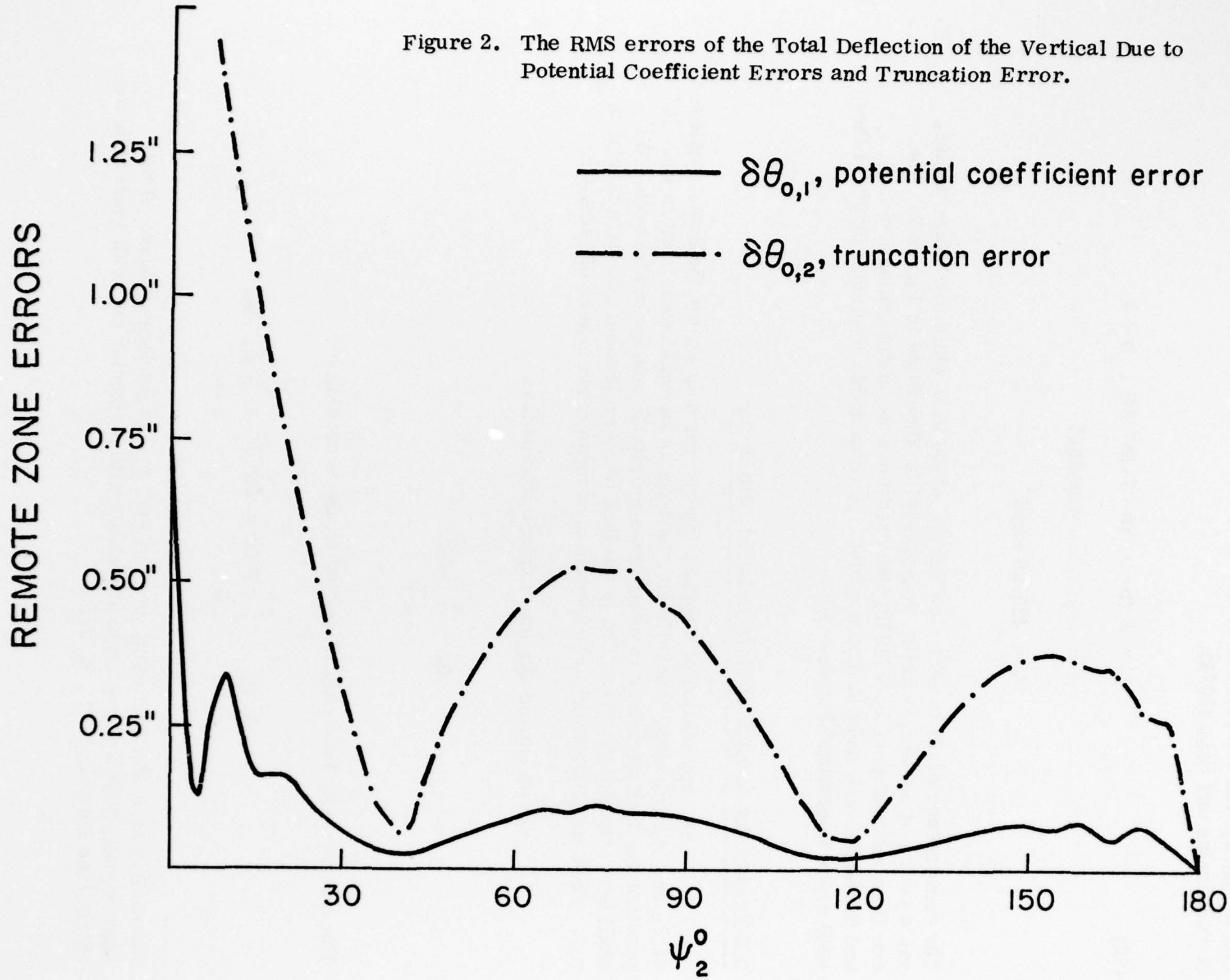

Figure 2. The RMS errors of the Total Deflection of the Vertical Due to Potential Coefficient Errors and Truncation Error.

(11) $$\beta_n = [P_{n-1}(\cos\psi_0) - P_{n+1}(\cos\psi_0)]/(2n+1)(1-\cos\psi_0)$$

where ψ_0 is the spherical radius of the cap. If ν is the block size, then ψ_0 is given by:

$$\psi_0 = \sqrt{\nu \sin\nu/\pi}$$

Now the error at each point when representing Δg by $\overline{\Delta g}$ is:

$$\delta g = \Delta g - \overline{\Delta g} = \sum_{n=2}^{\infty} (1-\beta_n)\,\Delta g_n$$

Following the derivation of formula (5) in Heiskanen and Moritz (1967, pp. 261-262), we obtain the following error propagation of δg outside the spherical distance ψ.

The error of each component ξ and η becomes:

$$\begin{Bmatrix}\delta\xi\\ \delta\eta\end{Bmatrix} = -\frac{1}{2G}\sum_{n=2}^{\infty} \bar{Q}_n(\psi) \begin{Bmatrix}\dfrac{\partial}{\partial\varphi}\\ \dfrac{1}{\cos\varphi}\dfrac{\partial}{\partial\lambda}\end{Bmatrix} \delta g$$

and the total RMS error is given by:

$$\delta\theta^2 = M\{\delta\xi^2 + \delta\eta^2\} =$$

$$= \frac{1}{4G^2}\sum_{n=2}^{\infty}\sum_{n'=2}^{\infty} \bar{Q}_n(\psi)\,\bar{Q}_{n'}(\psi)\, M\left\{\frac{\partial\delta g_n}{\partial\varphi}\frac{\partial\delta g_{n'}}{\partial\varphi} + \frac{1}{\cos^2\varphi}\frac{\partial\delta g_n}{\partial\lambda}\frac{\partial\delta g_{n'}}{\partial\lambda}\right\} =$$

$$= \frac{1}{4G^2}\sum_{n=2}^{\infty} \bar{Q}_n^2(\psi)\, n(n+1)\, M\{\delta g_n^2\}$$

where

$$\delta g_n = (1-\beta_n)\,\Delta g_n$$

and

$$M\{\delta g_n^2\} = (1-\beta_n)^2\, M\{\Delta g_n^2\} = (1-\beta_n)^2 \left(\frac{r_B}{r}\right)^{2(n+2)} c_n$$

Hence, by using Vening Meinesz' formula outside the spherical distance ψ, the following mean square error (due to neglecting more detailed data) is committed:

$$\delta\theta^2 = \frac{1}{4G^2}\sum_{n=2}^{\infty} \bar{Q}_n^2(\psi)\, n(n+1)(1-\beta_n)^2 \left(\frac{r_B}{r}\right)^{2(n+2)} c_n$$

Finally, the corresponding contribution from the zone σ_2 is given by:

(12) $$\delta\theta_{2,1}^2 = \frac{1}{4G^2}\sum_{n=2}^{\infty} [\bar{Q}_n^2(\psi_1) - \bar{Q}_n^2(\psi_2)]\, n(n+1)(1-\beta_n^2) \left(\frac{r_B}{r}\right)^{2(n+2)} c_n$$

As β_n approaches 0 for large n this formula suffers from the same slow convergence as (9) for small angles ψ_1. For $1^\circ \le \psi_1 \le 4^\circ$ (12) was expanded to $n = 5000$. For larger ψ_1, $n = 2000$ was found to be a sufficient degree of truncation ($\nu = 1^\circ$). The results are shown in Figure 3.

3.3 Errors Due to Inaccurate Gravity Material in σ_2

A constant error in Δg will not affect the deflections of the vertical, because there is no zero-order term present in ξ and η. Hence, the only error source that has to be considered is the random errors due to insufficient gravity data within each block to assure an accurate determination of the mean value. The propagation of this error is given by (3). We obtain approximately:

(13) $$\delta\theta_{2,2}^2 = \frac{1}{(4\pi G)^2}\sum_i m_i^2 \left[\iint_{\sigma_i} \frac{dS(\psi)}{d\psi}\, d\sigma\right]^2$$

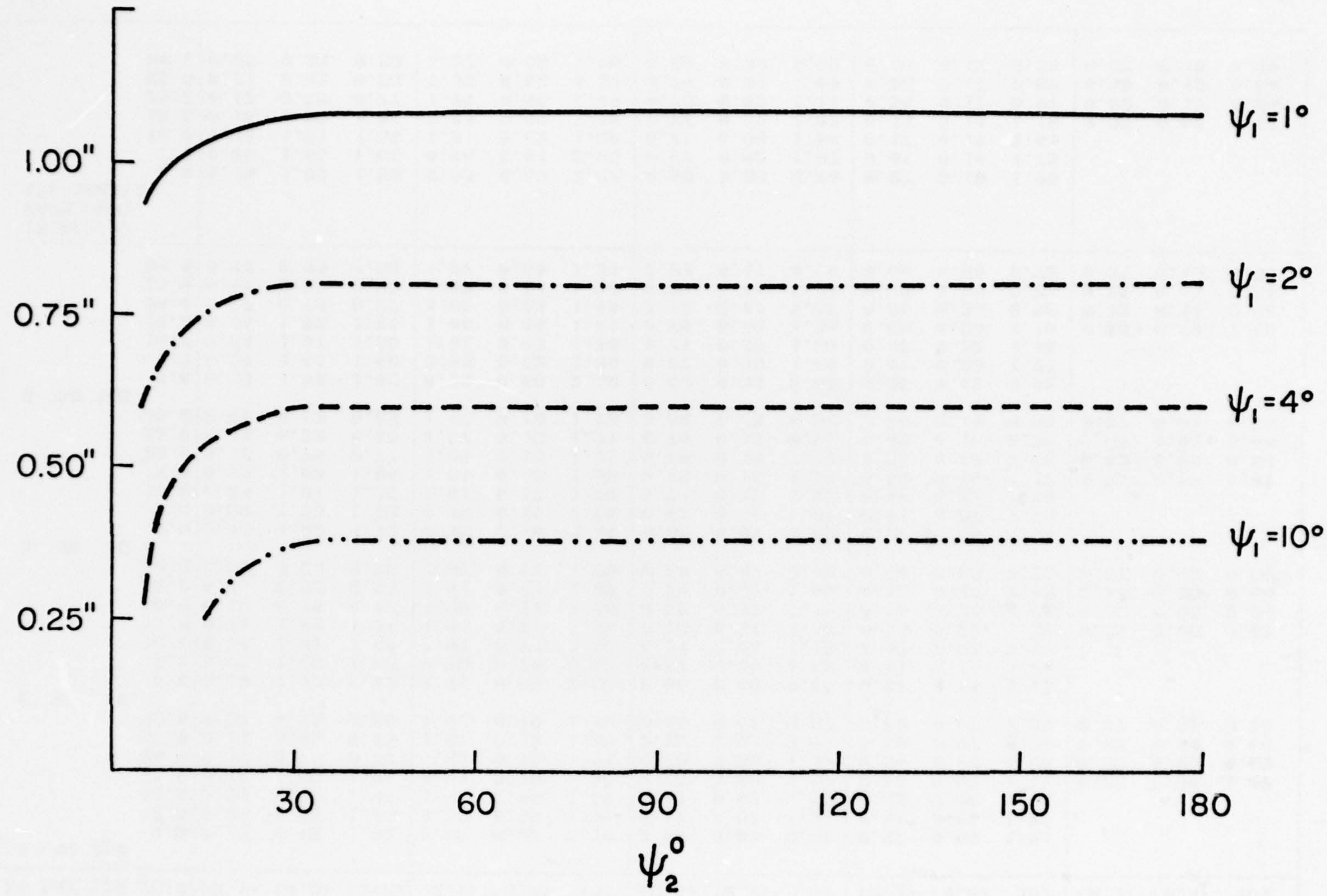

Figure 3. The RMS Error of theTotal Deflection of the Vertical Due to Lack of More Detailed Gravity Material than $1^\circ \times 1^\circ$ Mean Anomalies.

TABLE 2. SUMMATION OF DEFLECTION ERRORS FOR 4 SPECIFIC POINTS.
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POINT					PSI1=1 DEG.			PSI1=2 DEG.			PSI1=4 DEG.			PSI1=10 DEG.		
NO LAT LON	PSI2	(0,1)	(0,2)	(1+2)	(2,1)	(2,2)	TOT	(2,1)	(2,2)	TOT	(2,1)	(2,2)	TOT	(2,1)	(2,2)	TOT
1 40 270																
	5.0	0.13	1.92	1.92	0.93	0.07	2.14	0.60	0.04	2.02	0.27	0.02	1.94			
	7.5	0.29	1.65	1.68	0.98	0.09	1.94	0.67	0.07	1.81	0.41	0.06	1.73			
	10.0	0.34	1.31	1.35	1.01	0.09	1.69	0.71	0.07	1.53	0.47	0.06	1.43			
	15.0	0.16	1.03	1.04	1.04	0.10	1.48	0.75	0.08	1.29	0.53	0.07	1.17	0.25	0.04	1.07
	20.0	0.17	0.75	0.77	1.05	0.10	1.31	0.78	0.08	1.10	0.56	0.07	0.95	0.32	0.04	0.83
	25.0	0.11	0.52	0.53	1.07	0.10	1.20	0.79	0.08	0.96	0.58	0.07	0.79	0.35	0.04	0.64
	30.0	0.07	0.32	0.33	1.07	0.10	1.12	0.80	0.08	0.87	0.60	0.07	0.69	0.37	0.04	0.50
2 35 300																
	5.0	0.13	1.92	1.92	0.93	0.69	2.25	0.60	0.43	2.06	0.27	0.14	1.95			
	7.5	0.29	1.65	1.68	0.98	0.70	2.06	0.67	0.45	1.86	0.41	0.19	1.74			
	10.0	0.34	1.31	1.35	1.01	0.71	1.83	0.71	0.46	1.60	0.47	0.22	1.45			
	15.0	0.16	1.03	1.04	1.04	0.71	1.63	0.75	0.47	1.37	0.53	0.23	1.19	0.25	0.08	1.07
	20.0	0.17	0.75	0.77	1.05	0.71	1.48	0.78	0.47	1.19	0.56	0.23	0.98	0.32	0.08	0.84
	25.0	0.11	0.52	0.53	1.07	0.71	1.39	0.79	0.47	1.06	0.58	0.23	0.82	0.35	0.08	0.64
	30.0	0.07	0.32	0.33	1.07	0.71	1.33	0.80	0.47	0.98	0.60	0.23	0.72	0.37	0.08	0.50
3 50 15																
	5.0	0.13	1.92	1.92	0.93	0.16	2.14	0.60	0.09	2.02	0.27	0.04	1.94			
	7.5	0.29	1.65	1.68	0.98	0.16	1.95	0.67	0.11	1.81	0.41	0.07	1.73			
	10.0	0.34	1.31	1.35	1.01	0.17	1.70	0.71	0.12	1.53	0.47	0.09	1.44			
	15.0	0.16	1.03	1.04	1.04	0.18	1.48	0.75	0.13	1.29	0.53	0.10	1.17	0.25	0.05	1.07
	20.0	0.17	0.75	0.77	1.05	0.18	1.31	0.78	0.13	1.10	0.56	0.10	0.96	0.32	0.05	0.83
	25.0	0.11	0.52	0.53	1.07	0.18	1.21	0.79	0.13	0.96	0.58	0.10	0.79	0.35	0.05	0.64
	30.0	0.07	0.32	0.33	1.07	0.18	1.13	0.80	0.13	0.87	0.60	0.10	0.69	0.37	0.05	0.50
4 30 145																
	5.0	0.13	1.92	1.92	0.93	0.60	2.22	0.60	0.35	2.05	0.27	0.13	1.95			
	7.5	0.29	1.65	1.68	0.98	0.62	2.04	0.67	0.38	1.84	0.41	0.20	1.74			
	10.0	0.34	1.31	1.35	1.01	0.63	1.80	0.71	0.39	1.58	0.47	0.22	1.45			
	15.0	0.16	1.03	1.04	1.04	0.64	1.61	0.75	0.40	1.34	0.53	0.24	1.19	0.25	0.09	1.08
	20.0	0.17	0.75	0.77	1.05	0.64	1.45	0.78	0.41	1.17	0.56	0.24	0.98	0.32	0.10	0.84
	25.0	0.11	0.52	0.53	1.07	0.64	1.36	0.79	0.41	1.04	0.58	0.24	0.82	0.35	0.10	0.64
	30.0	0.07	0.32	0.33	1.07	0.64	1.29	0.80	0.41	0.96	0.60	0.24	0.72	0.37	0.10	0.50
TETA(2,2) FROM GROT. AND MORITZ																
	5.0	0.13	1.92	1.92	0.93	0.60	2.22	0.60	0.28	2.04	0.27	0.10	1.95			
	7.5	0.29	1.65	1.68	0.98	0.61	2.03	0.67	0.30	1.83	0.41	0.14	1.73			
	10.0	0.34	1.31	1.35	1.01	0.62	1.80	0.71	0.30	1.56	0.47	0.14	1.44			
	15.0	0.16	1.03	1.04	1.04	0.62	1.60	0.75	0.32	1.32	0.53	0.14	1.18	0.25	0.07	1.07
	20.0	0.17	0.75	0.77	1.05	0.62	1.44	0.78	0.32	1.14	0.56	0.17	0.97	0.32	0.10	0.84
	25.0	0.11	0.52	0.53	1.07	0.62	1.35	0.79	0.32	1.00	0.58	0.17	0.80	0.35	0.10	0.64
	30.0	0.07	0.32	0.33	1.07	0.62	1.28	0.80	0.32	0.92	0.60	0.17	0.70	0.37	0.10	0.50

where m_i is the standard error of Δg_i and σ_i is the area of block i. The evaluation of (13) requires that m_i is known for all blocks used in the determination of ξ_2 and η_2. It will therefore be very much dependent on the quality of the mean anomalies and will vary for each point of computation. In Table 2, we report the computations in four specific points. The computations are based on the $1° \times 1°$ anomaly information described in Rapp (1977). For unknown blocks m_i is set to 30 mgal. In this table we also show the error estimates of Groten and Moritz (1964). By assuming uniform errors of all Δg blocks, they arrived at the following formula:

$$\delta\theta^2_{2,2} = \frac{S}{8\pi(Gr)^2}\left[J(\psi_1) - J(\psi_2)\right] \tag{14}$$

where

$$J(\psi) = \int_{\psi}^{\pi} \left(\frac{dS(\psi)}{d\psi}\right)^2 \sin\psi \, d\psi$$

$S = 0.027\ r^2$ ($1° \times 1°$ blocks, 1 gravity profile inside each block). Formula (14) seems to give a reasonable approximation of $\delta\theta_{2,2}$ for $\psi_1 \geq 2°$, specially in areas with a poor gravity material (points 2 and 4).

In Table 2 we also report the sum of squares of the errors considered above (the error of the inner zone is not included). From these results we conclude that the RMS errors are significantly decreasing with an increasing angle ψ_2 all the way to 30°.

3.4 An Extended View

On the basis of the previous computation results, it is also of interest to study the errors when ψ_2 is extended beyond 30°. However, such an extension would include large areas where no $1° \times 1°$ gravity material exists today. In these computations, we will therefore assume that we have such a material with a uniform error of all blocks. Thus, we can apply formula (14) for the computation of $\delta\theta_{2,2}$. The resulting RMS errors are shown in Figure 4.

From Figures 2 and 4 we conclude that a dominating error source is the truncation of the spherical harmonic expansion. A significant minimum of the RMS error is obtained for $\psi_2 = 40°$. It is unreasonable to extend ψ_2 beyond this minimum.

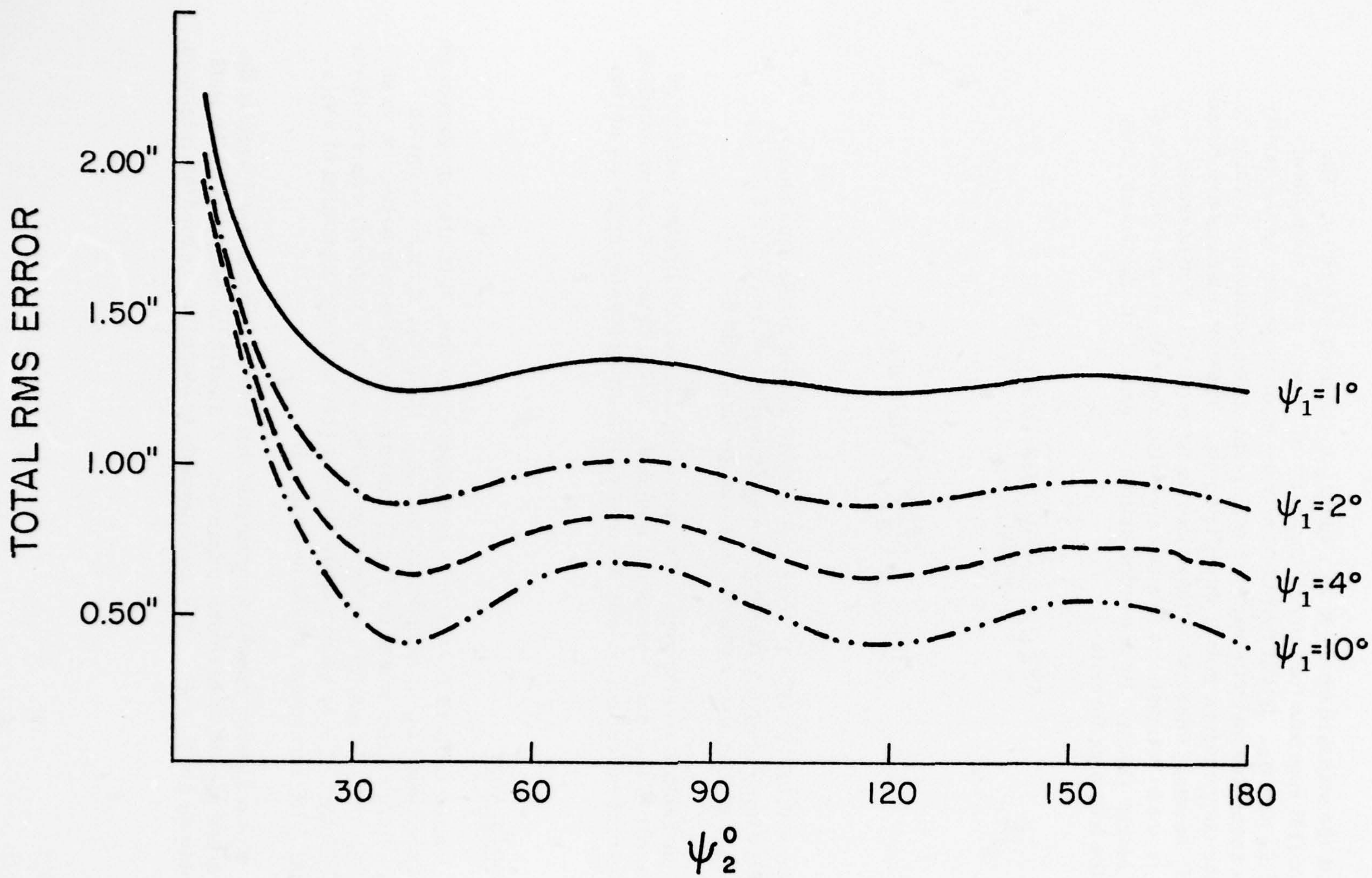

Figure 4. The Total RMS Error of the Deflection of the Vertical (θ)
(The Inner Zone Excluded).

3.5 The Inner Zone Error and the Total Error

The error of the inner zone depends on the quality and distribution of the observations in this area. To some extent it will also vary with the method used in the computations. By using least squares collocation with approximately 200 observations, Lachapelle (ibid.) estimated the errors of ξ_1 and η_1 to about 1".3. These errors were rather independent of the cap size ψ_1, which varied between 0°.7 and 1°.5 in the computations. Hence, by using this technique, the total RMS error of θ will be in the order of 2".2 for $\psi_1 = 1°$ and $\psi_2 = 40°$.

Kearsley (1976) applied the original Vening Meinesz' formula with the well-known subdivision into Rice Rings. The inner zone errors of ξ_1 and η_1 were about 0".3 for $\psi_1 \approx 1°$. Thus, a total RMS error of θ in the order of 1".3 (for $\psi_2 = 40°$) seems possible to achieve in an intensive determination of the inner zone contribution. Even though the same accuracy might be possible to obtain by using least squares collocation, there are practical limits of the number of observations in this method (Lachapelle, ibid., p. 6). Here we refer to $\delta\xi = \delta\eta = 0".3$ as the accuracy of Vening Meinesz' formula. Some total RMS errors based on the above estimates of $\delta\theta_1$ are given in Table 3.

Table 3. Total RMS Errors of the Deflections of the Vertical for $\psi_1 = 1°$

ψ_2	5°	10°	20°	30°	40°
Collocation $\delta\theta_1 = 1".84$	2".88	2".57	2".34	2".24	2".22
Vening Meinesz $\delta\theta_1 = 0".42$	2".26	1".85	1".50	1".35	1".32

Lachapelle (ibid.) compared predicted deflections of the vertical at some 169 astrogeodetic stations with the astrogeodetic deflection components. The RMS difference between the total deflections was 2".12. This result agrees fairly well with Table 3.

4. Summary and Conclusions

In this paper we have studied possible error sources for the determination of the deflections of the vertical by using a combined method. The investigation deals mainly with the errors generated in the intermediate and remote zone, while for the inner zone, some error figures have been adopted from earlier studies. The numerical results are shown in Tables 2 - 3 and Figures 2 - 4.

We assume that the only contribution to ξ and η from the remote zone is a spherical harmonic expansion to degree 16. The RMS truncation error of such a series was found considerably sensitive to changes in the distance of truncation, ψ_2 (see Figure 2). A local minimum ($\delta\theta_{0,2}$ = 0".06) is obtained for $\psi_2 = 40°$.

In the intermediate zone the gravity field was assumed to be represented by $1° \times 1°$ mean anomalies. The error due to lack of a more detailed gravity material is depicted in Figure 3. This error source can be diminished by using a more detailed subdivision of the blocks between the distances 1° and 10° from the computation point.

The error propagation of the mean anomaly errors ($\delta\theta_{2,2}$) is shown in Table 2 for four selected points. A reasonable approximation of these errors is obtained by using formula (14) according to Moritz and Groten.

The RMS sum of the errors of the intermediate and remote zone is given in Figure 4. A significant gain in accuracy is achieved by extending ψ_2 to at least 30°. A minimum is obtained for $\psi_2 = 40°$. Finally, by adding a representative error for the inner zone computation, the total RMS error was estimated (Table 3). If the inner zone is determined by the method of least squares collocation, the total RMS error will hardly be less than 2".2 ($\psi_1 = 1°$). Substituting this method with an accurate version of Vening Meinesz' formula, the final error might decrease to 1".3. Further improvements are expected for a refined subdivision of the intermediate zone blocks. In no case should the truncation distance (ψ_2) exceed 40°.

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